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Deconfinement at $N > 2$: $\mathrm{SU}(N)$ Georgi-Glashow model in $2+1$ dimensions

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Abstract: We analyse the deconfining phase transition in the $\mathrm{SU}(N)$ Georgi-Glashow model in $2+1$ dimensions. We show that the phase transition is second order for any N, and the universality class is different from the Z_N invariant Villain model. At large N the conformal theory describing the fixed point is a deformed $\mathrm{SU}(N)_1$ WZNW model which has $N-1$ massless fields. It is therefore likely that its self-dual infrared fixed point is described by the Fateev-Zamolodchikov theory of Z_N parafermions.

Keywords: Field Theories in Lower Dimensions, Confinement.

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Contents

1. Introduction

Recently [15] we have analysed in detail the deconfining phase transition in the SU(2) Georgi-Glashow (GG) model in $2+1$ dimensions. The mechanism of confinement in this model at zero temperature is due to the "plasma" of the monopole-instantons and is well understood [1]. The model is weakly interacting all the way up to the deconfining temperature, which allowed us to study the phase transition quantitatively. We found that taking into account the excitations of the heavy charged particles was crucial for the correct description of the transition. The transition is associated with the restoration of the magnetic Z_2 symmetry [2, 3] in accordance with general arguments of [4]. The universality class of the transition was found to be 2d Ising.

Whereas for SU(2) gauge theory there is overwhelming consensus that the transition should be in the universality class of the Ising model, the situation is much less clear for large N. The point is that for $N > 3$ one can write down different 2d spin models, and they have different critical behaviour. For example the N-state Potts models have first order phase transition for $N > 4$ [5], while Villain models have second order transition which is of the BKT type, and is thus in the universality class of U(1) [6]. Whether the transition in the SU(N) gauge theory is similar to either one of those, is an open interesting question.

In this paper we consider a general Georgi-Glashow type SU(N) gauge theory, where at zero temperature the gauge group is spontaneously broken to $U^{N-1}(1)$. Just like the SU(2) GG model, the theory is weakly interacting. At zero temperature it is confining, and the monopole "plasma" description of confinement has long been known [7]. It has also been studied from the point of view of magnetic Z_N symmetry in [8].

Our main finding is that the transition in the model is second order, and is distinct from that of Villain model. Although we are unable to identify the fixed point theory with a known two-dimensional conformal theory, we argue that the relevant model at large N must be a deformation of a theory with a large value of the UV central charge $c = O(N)$, which may be SU$(N)_1$ WZNW model.

The paper is structured as follows. In section 2 we describe the model as well as the monopole and magnetic symmetry based approaches to its low energy dynamics. In section 3 we derive the dimensionally reduced model relevant for the study of the phase transition, and discuss the role of the heavy charged particles. In section 4 we study the transition with the help of the renormalization group analysis in the reduced theory. We show that the RG equations have a self dual infrared fixed point. We explain why the GG model close to the transition does not behave like Villain model, even in the range of parameters where one might expect it to do so. In section 5 we point out to similarities between the behaviour of some quantities in the GG model close to criticality and in the hot Yang-Mills theory. Finally in section 6 we discuss our results.

2. The model

We consider the SU(N) gauge theory with scalar fields in the adjoint representation in $2+1$ dimensions.

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{tr}\,D_\mu\Phi D^\mu\Phi - V(\Phi)\,, \tag{2.1}$$

where

$$\begin{aligned} A_\mu &= A_\mu^a T^a\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]\,,\\ \Phi &= \Phi^a T^a\,, \qquad D_\mu\Phi = \partial_\mu\Phi + g[A_\mu, \Phi]\,. \end{aligned} \tag{2.2}$$

T^a are traceless hermitean generators of the SU(N) algebra normalised as $\mathrm{tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$.

Depending on the form of the Higgs potential, there can be different patterns of gauge symmetry breaking. Since most of the details of the potential are unimportant for our purposes, we will not specify it except for restricting it to the region of the

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parameter space where classically the gauge symmetry is broken to the maximal torus

$$\mathrm{SU}(N) \longrightarrow \mathrm{U}(1)^{N-1} . \tag{2.3}$$

We also restrict ourselves to weakly coupled regime, which means that the ratios M_W/g^2 are large for *all* $N^2 - N$ massive W-bosons.

2.1 The perturbative spectrum

To characterise the perturbative spectrum of the theory it is convenient to use the Cartan-Weyl basis $(H^i, E^{\vec{\alpha}})$, where H^i generate the Cartan subalgebra which is of the dimension of rank of $\mathrm{SU}(N)$: $r = N - 1$.

$$[H^i, H^j] = 0 , \qquad i, j \in [1, 2, \ldots, N-1] \tag{2.4}$$

and $E^{\vec{\alpha}}$ are the $N(N-1)$ ladder operators which satisfy

$$\begin{aligned} [H^i, E^{\vec{\alpha}}] &= \alpha^i E^{\vec{\alpha}} , \\ [E^{\vec{\alpha}}, E^{\vec{\beta}}] &= N_{\vec{\alpha},\vec{\beta}}\, E^{\vec{\alpha}+\vec{\beta}} , \qquad \text{if } \vec{\alpha} + \vec{\beta} \text{ is a root} , \\ &= 2\vec{\alpha} \cdot \vec{H} , \qquad \text{if } \vec{\alpha} = -\vec{\beta} . \end{aligned} \tag{2.5}$$

The $N-1$-dimensional root vectors $\vec{\alpha} = (\alpha^1, \alpha^2, \ldots, \alpha^{N-1})$ form the dual Cartan subalgebra. There are obviously $N(N-1)$ such vectors corresponding to $\dim(\mathrm{SU}(N)) - \mathrm{rank}(\mathrm{SU}(N))$ but only $N - 1$ of them are linearly independent. The non-vanishing inner products in the Cartan-Weyl basis read as

$$\mathrm{tr}(H^i, H^j) = \frac{1}{2}\delta^{ij} , \qquad \mathrm{tr}(E^{\vec{\alpha}}, E^{\vec{\beta}}) = \frac{1}{2}\delta^{\vec{\alpha},-\vec{\beta}} . \tag{2.6}$$

At the classical level $N-1$ gauge group generators are unbroken, which we choose to correspond to (H^i). Therefore classically there are $N - 1$ massless photons and $N(N-1)$ charged massive W-bosons.

Our Weyl basis is chosen in such a way that the Higgs VEV is diagonal. Since the matrix Φ is traceless, there are $N - 1$ independent eigenvalues. In terms of the $N - 1$-dimensional vector $\vec{h} = (h_1, h_2, h_3, \ldots, h_{N-1})$[1] we have

$$\langle\Phi\rangle = \vec{h} \cdot \vec{H} , \qquad A_\mu = \vec{A}_\mu \cdot \vec{H} + \sum_{\vec{\alpha}} A_\mu^{\vec{\alpha}} E^{\vec{\alpha}} . \tag{2.7}$$

For concreteness let us choose the following basis for the Cartan subalgebra;

$$\begin{aligned} H_1 &= \frac{1}{2}\,\mathrm{diag}(1, -1, 0, \ldots, 0) , \qquad H_2 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}(1, 1, -2, 0, \ldots, 0) , \\ \cdots \qquad H_{N-1} &= \frac{1}{\sqrt{2N(N-1)}}\,\mathrm{diag}(1, 1, 1, \ldots, 1, -(N-1)) . \end{aligned} \tag{2.8}$$

[1] For concreteness we order these numbers $h_1 > h_2 > \cdots > h_{N-2} > h_{N-1}$, which also breaks the discrete Weyl group.

As long as $\vec{h} \cdot \vec{\alpha} \neq 0$ for all roots, the gauge symmetry is maximally broken. The masses of the W-bosons can be read off from the second term in the lagrangian

$$
\begin{aligned}
g^2 \operatorname{tr}[A_\mu, \Phi]^2 &= \frac{g^2}{2} \sum_{\vec{\alpha},i,j} A_\mu^{\vec{\alpha}} A_\mu^{-\vec{\alpha}} h_i h_j \alpha^i \alpha^j \,, \\
\Longrightarrow M_{\vec{\alpha}} &= g|\vec{h} \cdot \vec{\alpha}| \,.
\end{aligned} \tag{2.9}
$$

The W-bosons corresponding to the $N-1$ simple roots $\vec{\beta}_i$, $i = 1, \ldots, N-1$ (arbitrarily chosen set of linearly independent roots) can be thought of as fundamental, in the sense that the quantum numbers and the masses of all other W-bosons are obtained as linear combinations of those of the fundamental W-bosons. These charges and masses are

$$
\vec{Q}_{\vec{\beta}} = g\vec{\beta} \,, \qquad M_{\vec{\beta}} = g\vec{h} \cdot \vec{\beta} \,. \tag{2.10}
$$

As an example consider the case of SU(3) broken down to U(1) × U(1). There are 6 massive W-bosons. The simple roots can be taken as

$$
\vec{\beta}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \qquad \vec{\beta}_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \tag{2.11}
$$

The remaining non-simple positive root is

$$
\vec{\alpha}_3 = \vec{\beta}_1 - \vec{\beta}_2 = (1, 0) \,. \tag{2.12}
$$

The other three roots are $-\vec{\beta}_i$, $-\vec{\alpha}_3$. The masses of corresponding W-bosons are

$$
M_{W_1} = \frac{g}{2}(h_1 + \sqrt{3}h_2) \,, \qquad M_{W_2} = \frac{g}{2}(h_1 - \sqrt{3}h_2) \,, \qquad M_{W_2} = gh_1 \tag{2.13}
$$

for $h_1 > \sqrt{3}h_2$. Observe that if $h_2 = 0$, two of the masses become degenerate. In this case SU(3) is still broken down to U(1)×U(1) since all three masses are non vanishing but the spectrum is invariant under an additional Z_2 symmetry. This Z_2 symmetry is the charge conjugation with respect to the charge H_2, which interchanges the roots $\vec{\beta}_1$ and $\vec{\beta}_2$. In general though, this charge conjugation symmetry is broken by the VEV of Higgs [11].

2.2 The monopole-instantons and the Polyakov effective lagrangian

Non-perturbatively the most important contributions in the theory are due to the monopole-instantons. Those are classical, stable, finite action solutions of the euclidean equations of motion arising due to the non-trivial nature of the second homotopy group of the vacuum manifold ($\Pi_2(\mathrm{SU}(N)/\,\mathrm{U}(1)^{N-1}) = Z^N$). The magnetic field of such a monopole is long range.

$$
B_\mu = \frac{x^\mu}{4\pi r^3} \vec{g} \cdot \vec{H} \,. \tag{2.14}
$$

The $N-1$-dimensional vectors $\vec{g}$ are determined by the non-abelian generalisation of the Dirac quantisation condition [13, 12]

$$e^{ig\vec{g}\cdot\vec{H}} = I\,. \tag{2.15}$$

Solution of this quantisation condition are

$$\vec{g} = \frac{4\pi}{g}\sum_{i=1}^{N-1} n_i \vec{\beta}_i^{*}\,, \tag{2.16}$$

where $\vec{\beta}^*$ are the dual roots defined by $\vec{\beta}^* = \vec{\beta}/|\vec{\beta}|^2$. We will be working with roots normalised to unity, and thus $\vec{\beta}^* = \vec{\beta}$. The integers n_i are elements of the group Π_2 [9]. The monopoles which have the smallest action correspond to roots taken once. The action of these monopoles in the BPS limit is

$$M_{\vec{\alpha}} = \frac{4\pi}{g}\vec{h}\cdot\vec{\alpha} = \frac{4\pi M_{W_{\vec{\alpha}}}}{g^2}\,. \tag{2.17}$$

Just like with W-bosons we can think of monopoles corresponding to simple roots as fundamental ones with magnetic charges and action

$$\vec{g_i} = \frac{4\pi}{g}\vec{\beta}_i\,, \qquad M_i = \frac{4\pi}{g}\vec{h}\cdot\vec{\beta}_i\,. \tag{2.18}$$

For example, in the case of SU(3) (see eqs. (2.11) and (2.13)) the monopole action spectrum (in the BPS limit) is

$$M_1 = \frac{2\pi}{g}(h_1+\sqrt{3}h_2)\,, \qquad M_2 = \frac{2\pi}{g}(h_1-\sqrt{3}h_2)\,, \qquad M_3 = \frac{4\pi}{g}h_1\,. \tag{2.19}$$

The effect of these monopoles is to impart finite mass to all the perturbatively massless "photons". The derivation of the effective lagrangian follows exactly the same lines as the original derivation of Polyakov for the SU(2) theory [1]. The resulting low energy effective theory is written in terms of the $N-1$ component field, $\vec{\eta}$, with the following lagrangian [7, 8]

$$\mathcal{L}_{\text{eff}} = \frac{g^2}{32\pi^2}(\partial_\mu\vec{\eta})^2 + \sum_\alpha \frac{M_\alpha^2 g^2}{16\pi^2}\exp(i\vec{\alpha}\cdot\vec{\eta})\,. \tag{2.20}$$

The sum is over all $N(N-1)$ non-vanishing roots. The potential induced by the monopoles is proportional to the monopole fugacity

$$M_\alpha^2 = \frac{16\pi^2\xi_\alpha}{g^2}\,, \qquad \xi_\alpha = \text{constant}\;\frac{M_{W_\alpha}^{7/2}}{g}e^{\frac{-4\pi M_{W_\alpha}}{g^2}\epsilon(M_H/M_W)}\,. \tag{2.21}$$

$\epsilon(M_H/M_W)$ is such that $1 \le \epsilon \le 1.787$ [10], and $\epsilon(\infty) = 1$.

The photons at weak coupling are obviously much lighter than the W-bosons and thus are the only relevant degrees of freedom in the low energy sector.

2.3 The magnetic Z_N symmetry

The global symmetry structure is very important for the understanding of the deconfining transition. The relevant symmetry in the present model is the magnetic Z_N symmetry. We now wish to explain how this symmetry is implemented in the effective low energy lagrangian. Our discussion parallels the SU(2) case [15].

The order parameter of the magnetic symmetry is the set of magnetic vortex operators V_i, $i = 1, \ldots, N-1$. These operators were constructed explicitly in [3]. These operators carry magnetic fluxes of the $N-1$ U(1) abelian magnetic fields. The defining commutation relation for V_i is

$$[V_i(x), \vec{B}(y)] = -\frac{4\pi}{g}\vec{w}_i V_i(x)\delta^2(x-y)\,. \tag{2.22}$$

Here $\vec{B}$ is the $N-1$-dimensional vector of magnetic fields,[2] whose j-th component is the projection of the non-abelian field strength onto the direction of the Cartan subalgebra generator H_j, and $\vec{w}_j$ are $N-1$ weight vectors of SU(N). The choice of the $N-1$ out of N weight vectors is arbitrary. Change in this choice will lead to the redefinition of the vortex operators such that the new operators will be products of the old ones and their conjugates. It is always possible to choose these weights so that together with the "fundamental" roots β_i they satisfy the relation

$$\vec{w}_i\vec{\beta}_j = \frac{1}{2}\delta_{ij}\,. \tag{2.23}$$

The flux eigenvalues in eq. (2.22) are dictated by the requirement of the locality of the vortex operators and is analogous to the Dirac quantisation condition. The explicit form of the vortex operators in terms of the field η in eq. (2.20) is

$$V_i(\vec{x}) = \frac{g}{\sqrt{8\pi}}\, e^{i\chi_i} \tag{2.24}$$

with

$$\chi_i = \vec{w}_i \cdot \vec{\eta} \Longrightarrow \vec{\eta} = 2\sum_i \vec{\beta}_i\chi_i\,. \tag{2.25}$$

The effective lagrangian can be written as a non-linear σ-model in terms of V_i as

$$\mathcal{L}_{\text{eff}} = \frac{N-1}{2}\sum_{i,j} A_{ij}\frac{1}{V_k^* V_k}(V_i^*\partial_\mu V_i)(V_j\partial_\mu V_j^*) + \\ +\lambda\sum_i\left(V_iV_i^* - \frac{g^2}{8\pi^2}\right)^2 + \sum_\alpha k_\alpha \prod_i V_i^{2i\vec{\alpha}\cdot\vec{\beta}_i} \tag{2.26}$$

with $\lambda \to \infty$. The matrix $A_{ij} = 2\vec{\beta}_i^* \cdot \vec{\beta}_j$ depends on the choice of the fundamental roots. With the conventional choice of positive roots, where $\vec{\beta}_i\vec{\beta}_j = -1/2$, $i \neq j$, it is

[2] Note that these magnetic fields can be constructed in gauge invariant way from the non-abelian field strengths and the Higgs field, see [3].

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the Cartan matrix of the Lie algebra. All its diagonal elements are equal to 2, while all its off diagonal elements equal to -1. We will find it however more convenient in the following to use a different set of fundamental roots, for which $\vec{\beta}_i\vec{\beta}_j = 1/2$, $i \neq j$. Such a choice is always possible for any SU(N). For this choice of roots the off-diagonal matrix elements of A_{ij} are all equal to 1.

For SU(3) we have

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \tag{2.27}$$

and the effective lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} = {} & \partial_\mu V_1 \partial_\mu V_1^* + \frac{8\pi^2}{g^2} V_1^* \partial_\mu V_1 V_2 \partial_\mu V_2^* + \partial_\mu V_2 \partial_\mu V_2^* + \\ & + \xi_1 (V_1 V_2^* + \text{c.c}) + \xi_2 (V_1^2 V_2 + \text{c.c}) + \xi_3 (V_1 V_2^2 + \text{c.c})\,. \end{aligned} \tag{2.28}$$

The magnetic Z_N symmetry has an obvious and simple representation in this effective lagrangian as $V_i \to \exp\{2\pi i n/N\} V_i$.

As long as only small fluctuations of the phase fields χ_i are important, the lagrangian eq. (2.28) is equivalent to the eq. (2.20). Thus at low temperature the descriptions based on these lagrangians are equivalent. The difference appears only when the phase nature of χ_i plays a role. Indeed, since χ_i are treated in eq. (2.28) as phases, dynamically one allows configurations in which these phases have non-trivial winding. On the other hand in eq. (2.20) such configurations cost infinite amount of energy. As discussed in detail in [3, 15] the winding configurations correspond to the heavy W-bosons. In fact the explicit relation between the vorticity of the fields V_i and the electric charges is given by [3]

$$\frac{1}{g} \vec{w}_i \vec{Q} = \frac{1}{4\pi} \oint_{C \to \infty} dl_\mu \partial_\mu \chi^i_\mu\,. \tag{2.29}$$

Thus the difference between the two lagrangians is important whenever the physics of the W-bosons plays an important role. We have seen in the case of the SU(2) theory that W's are indeed important near the phase transition temperature. The same turns out to be true for arbitrary SU(N). We thus have to be careful to treat the W-bosons properly in the transition region. In the next section we will set up this treatment.

3. The reduced theory

Throughout this paper we are working in the weak coupling regime and thus the photon masses in eq. (2.20) are exponentially small. Thus already at very low temperature ($T \propto M_\alpha$) one can use the dimensionally reduced version of the theory, since all the thermal modes are significantly heavier than the zero Matsubara frequency mode. Since the critical temperature for the deconfining transition is of order g^2 (see [15]), we can safely use dimensional reduction close to the transition. The zero Matsubara frequency sector is described by the *two*-dimensional lagrangian

$$\mathcal{L}_{\rm eff} = \frac{g^2}{32\pi^2 T}(\partial_\mu \vec{\eta})^2 + \sum_\alpha \frac{M_\alpha^2 g^2}{16\pi^2 T} \exp(i\vec{\alpha}\cdot\vec{\eta})\,. \tag{3.1}$$

However, as we noted before, our description should include W-bosons, and so the fields η should be treated here as phases with periodicity appropriate to eq. (2.25). In fact the lagrangian also has to be augmented by a four derivative "Skyrme" term, which fixes the energy of the winding states to be equal to the masses of W-bosons [15]. We can however simplify things further, by noting that the density of W-bosons at criticality is exponentially small due to the Boltzmann factor suppression. Thus W's can be treated in the dilute gas approximation in the same way as was done in [15]. To do this explicitly we first have to understand how to write partition function in the presence of one W-boson of a particular type.

Let us first consider a W-boson corresponding to one of the fundamental roots β_k. Using eqs. (2.10), (2.23) and (2.29) we see that this W-boson corresponds to unit vorticity of the field V_k and zero vorticity of all other fields V_j, $j\neq k$. To create such a vortex in the path integral we must introduce an external "current"which forces the discontinuity of the field χ_k

$$\chi_k = \chi_k + 2\pi\,. \tag{3.2}$$

The partition function in the presence of one W-boson is thus

$$Z = \int D[\chi(x)] \exp\left\{ - \int d^2y \frac{g^2}{16\pi^2 T} \sum A_{ij}(\partial_\mu\chi_i - J^i_\mu(y,x))(\partial_\mu\chi_j - J^j_\mu(y,x)) + \right.$$
$$\left. + \sum_\alpha \zeta_\alpha \cos\left(i2\sum_i \vec{\alpha}\cdot\vec{\beta}_i\chi_i\right)\right\}$$

with

$$J^i_\mu(y,x) = 2\pi\delta_{ik} n_\mu(y)\delta(y\in C_x) \tag{3.3}$$

with C_x a curve that starts at the location of the vortex (the point x), and goes to infinity, and n_μ is the unit normal to this curve. The insertion of this current forces the normal derivative of χ_k to diverge on curve C, so that χ_k jumps by 2π across C. Since in the rest of the space χ_k is smooth, the path integral is dominated by a configuration with unit vorticity of χ_k.[3]

[3] Note that even though J^i_μ explicitly depends on the curve C_x, the partition function itself does not, since changing the integration variable $\chi_i(x)\to\chi_i(x)+2\pi$, $x\in S$ where the boundary of S is $C_x - C'_x$ is equivalent to changing $C-x$ into C'_x in the definition of the current.

The path integral eq. (3.3) differs from the partition function in the vacuum sector by the linear term in the lagrangian

$$-\frac{g^2}{4\pi^2 T}\sum_{i,j}\int d^2y\,\vec{\beta}_i\cdot\vec{\beta}_j\,\partial_\mu\chi_i J^j_\mu = -\frac{g^2}{4\pi T}\int_{C_x} dx_\mu\epsilon_{\mu\nu}\,\partial_\nu\vec{\beta}_k\cdot\vec{\eta}\,. \tag{3.4}$$

Defining in the standard way the dual field $\tilde{\eta}$,

$$i\partial_\mu\vec{\tilde{\eta}} = \epsilon_{\mu\nu}\partial_\nu\vec{\eta} \tag{3.5}$$

we can recast the contribution of this particular W-boson in the form of the following extra term in the lagrangian

$$-i\frac{g^2}{4\pi T}\vec{\beta}_k\cdot\vec{\tilde{\eta}}\,. \tag{3.6}$$

This procedure can be repeated for W-boson corresponding to an arbitrary root α with the only difference that in eq. (3.6), the root β_k is replaced by the root α. To create several W-bosons one just inserts the external current which is the sum of the currents creating individual W's.

Dilute ensemble of such objects with small fugacities μ_α is then given by

$$Z = \prod_\alpha\sum_{n,m}\frac{1}{n!}\frac{1}{m!}\mu_\alpha^{n+m}\int\prod_i dx_i\prod_j dy_j Z(x_i,y_j)\,. \tag{3.7}$$

The summation over the number of W's can be easily performed, see [15]. The result is the partition function with the lagrangian

$$\mathcal{L}_{\rm eff} = \frac{g^2}{32\pi^2 T}(\partial_\mu\vec{\eta})^2 + \sum_\alpha\zeta_\alpha\exp(i\vec{\alpha}\cdot\vec{\eta}) + \sum_\alpha\mu_\alpha\exp\left(i\frac{g^2}{4\pi T}\vec{\alpha}\cdot\vec{\tilde{\eta}}\right) \tag{3.8}$$

with summation in both terms going over all non-vanishing roots of SU(N). The coefficients μ_α are proportional to the fugacities of the corresponding W-bosons

$$\mu_\alpha \propto \exp\left\{-\frac{M_{W_\alpha}}{T}\right\}. \tag{3.9}$$

Equation (3.8) is the dimensionally reduced theory which we will now use to study the phase transition.

4. The phase transition

4.1 Monopoles versus charges

To study the phase transition we may first attempt to disregard the W-boson induced term in the effective lagrangian. If we do that, we are back to the theory eq. (3.1). This theory is easily analysed. The first interesting thing about it is that since the group is simply laced (all the roots are of unit length) the anomalous dimensions of all the interaction terms are equal. The scaling dimension of all the monopole induced terms is

$$\Delta_M = \frac{4\pi T}{g^2}\,. \tag{4.1}$$

This immediately tells us that at the temperature

$$T_{\rm BKT} = \frac{g^2}{2\pi} \tag{4.2}$$

all these interactions become irrelevant. Thus at $T_{\rm BKT}$ one expects the Berezinsky-Kosterlitz-Thouless transition to take place. Above this temperature the infrared behaviour of the theory is that of $N-1$ free massless particles. Note that $T_{\rm BKT}$ does not depend on the number of colours N. If the picture just described where true, the universality class of the phase transition would be that of $U^{N-1}(1)$.

This of course is exactly the same situation as encountered in [14] in the SU(2) case. Again just like in SU(2) case this conclusion is incorrect due to the contribution of the W-bosons. To see this it is simplest to ask what would happen at high temperature if there were no monopole contributions at all. This amounts to studying eq. (3.8) with $\xi_\alpha = 0$. This theory describes non-compact electrodynamics with $N-1$ photons and the spectrum of charged particles given by eq. (2.10). This limit is again simple to understand, since the theory is exactly dual to the theory with monopoles and without charges. The scaling dimensions of all the W induced perturbations are equal and are given by

$$\Delta_W = \frac{g^2}{4\pi T}\,. \tag{4.3}$$

Thus the perturbations are irrelevant at low temperature, but become relevant at

$$T_{\rm NC} = \frac{g^2}{8\pi}\,. \tag{4.4}$$

Since $T_{\rm NC} < T_{\rm BKT}$ this tells us that we can not neglect the effects of charges at criticality. The story of SU(2) exactly repeats itself. Even the value of the temperature at which the scaling dimensions of the charge- and monopole induced perturbations are equal does not depend on N.

We expect therefore that the actual transition temperature is

$$T_C = \frac{g^2}{4\pi} \tag{4.5}$$

at which point all perturbations have the same scaling dimension. This expectation is confirmed by the renormalization group analysis.

4.2 Renormalization group analysis

The renormalization group equations for the theory eq. (3.8) were studied in [16]. In general the equations are quite complicated due to the cross correlations between different operators. For this reason the space of parameters of the theory has to be enlarged if one wants to study the flow whose UV initial condition is provided by eq. (3.8) with arbitrary values of fugacities. However there is one simple case, that is when the initial condition is such that all the monopole fugacities are equal $\xi_{\alpha_i} = \xi_{\alpha_j} = \xi$, and all the charge fugacities are equal $\mu_{\alpha_i} = \mu_{\alpha_j} = \mu$. This initial condition is stable under the RG flow. On this subspace the RG equations, written in terms of the scaled temperature $t = 4\pi T/g^2$ and dimensionless fugacities, read

$$\begin{aligned}
\frac{\partial t}{\partial \lambda} &= 2\pi^2 N t(\mu^2 - \zeta^2)\,, \\
\frac{\partial \mu}{\partial \lambda} &= \left(2 - \frac{1}{t}\right)\mu - 2\pi(N-2)\mu^2\,, \\
\frac{\partial \zeta}{\partial \lambda} &= (2-t)\zeta - 2\pi(N-2)\zeta^2\,.
\end{aligned} \tag{4.6}$$

These equations have exactly the property reflecting our previous discussion. That is the points $t = 2$, $\mu = 0$ and $t = 1/2$, $\xi = 0$ are both unstable. The stable IR fixed point is

$$t_o = 1\,, \qquad \mu_0 = \zeta_0 = \frac{1}{2\pi(N-2)}\,. \tag{4.7}$$

One can in fact easily check that in the three-dimensional space of couplings t, ξ and μ this point has two attractive and one repulsive direction. This is precisely what one expects from the IR fixed point located on the critical surface, the two attractive directions being the tangential directions to the surface.

The RG equations have an obvious duality symmetry, $\mu \to \xi$, $t \to 1/t$. This is the reflection of the transformation $\eta \to \tilde{\eta}$ on the level of the lagrangian eq. (3.8). The points $t = 1$, $\mu = \xi$ are symmetric under duality, and this ensures existence of a self dual fixed point. This is important, since the exact position of the fixed point is scheme dependent. Its existence however is assured by the duality symmetry.

What is the nature of this fixed point? For $N = 2$ we were able in [15] to fermionize the fixed point theory and show explicitly that it is equivalent to one massless Majorana fermion. We are not able to perform a similar analysis for arbitrary N. There are however several comments that we would like to make. Phase transitions in Z_N invariant spin models have been studied quite extensively. A nice recent discussion of the situation is given in [17]. One considers a spin model of one phase field θ with a symmetry breaking term of the type $h\,\cos\{N\theta\}$ which breaks the U(1) symmetry down to Z_N. When the coefficient h of this symmetry breaking term is large, the model resembles Potts model and thus (for $N > 4$) has a first order phase transition. When the breaking is small on the other hand, the behaviour is similar to the Villain model: the system undergoes two BKT type transitions with a massless U(1) symmetric phase at intermediate temperatures. At some particular "tricritical" value of h the massless phase shrinks to a point and it comes together with the first order transition line. This tricritical point is self-dual and is described by a conformal Z_N invariant parafermionic theory with the central charge $c = 2(N-1)/(N+2)$ introduced in [18]. In this type of models therefore generically one expects either the first order transition or a pair of BKT transitions with the massless phase in between. The tricritical behaviour is special and requires fine tuning of the parameters. This is indeed also the prevailing general expectation for the order of the transition in $2+1$-dimensional gauge theories at large N: either first order or Villain type U(1) invariant behaviour.

In fact we find our model in a completely different situation. The transition is not first order, and there is no U(1) invariant massless phase. We stress that within the RG flow eq. (4.6) the IR fixed point eq. (4.7) has two attractive directions. This means that it governs the IR behaviour of the points which lay on two-dimensional critical surface in the three-dimensional parameter space, and is therefore generic. This by itself does not preclude that this fixed point is the same as the parafermionic Z_N theory of [18]. If this is the case, it is quite interesting, since the point which appeared as "tricritical" from the point of view of usual spin models is in fact generic from the point of view of the 3D gauge theories. At present we can not prove that our critical point is described by the parafermionic theory but let us present some arguments supporting this conjecture. The point is that, as opposed to models considered in [17] our lagrangian eq. (3.8) describes a theory of $N-1$ light fields. The theory of $N-1$ free massless fields have the UV central charge $c_{\mathrm{UV}} = N-1$. However this CFT is deformed by the monopole and W — induced perturbations and flows to a different IR fixed point. However let us note that the central charge $c = N-1$ is precisely the central charge of the $\mathrm{SU}(N)_1$ WZNW model. The Ising (i.e. $c = 1/2$) model is the lowest among the minimal models with Virasoro (i.e. W_2) symmetry. The highest model of this class is $c = 1$ model (one free field) which is precisely $\mathrm{SU}(2)_1$ WZNW model. When the $c = 1$ model was deformed by the monopole and W-boson operators the central charge was reduced — and the resulting IR theory was Ising.

Now, Z_N parafermions with $c = 2(N-1)/N+2$ are the lowest minimal models with W_N symmetry — and the highest is $\mathrm{SU}(N)_1$ (for more information about parafermions see for example [19] and references therein) which can be described in terms of $N-1$ massless fields. Thus if the theory in the UV describes $N-1$ massless fields and has W_N symmetry, it is quite possible that result of the relevant (monopole $+W$) deformation is a self-dual critical point. It is indeed known that the Z_N parafermion theory is the self-dual model with W_N symmetry. The fact that the central charge (and thus the effective number of degrees of freedom) is reduced in the process of the flow towards IR is of course in complete accord with Zamolodchikov's C-theorem. It is therefore possible that the IR fixed point that describes the universality class of the GG model is the conformal Z_N parafermion theory.

Analysis of [16], although admittedly incomplete also supports the expectation that we do not have Villain picture. In fact it is the presence of the large number of fields that drives our theory away from the Villain behaviour as we will now explain.

4.3 Why not Villain?

The RG equations (4.6) were derived for the situation where all W-bosons have equal masses (all fugacities μ_α are equal). One can wonder what happens if this is not so. In particular imagine an extreme situation, where some W-bosons are light relative to the others, so that large monopole fugacities make all phase fields χ_i (or components of $\vec{\eta}$) but one relatively heavy. In this case at zero temperature the theory seems to have only one light degree of freedom. This situation is as close as it can be to the spin systems with one phase field, and one may expect that in this region of parameter space the finite temperature behaviour will be similar to that in the Villain model. The appearance of the intermediate massless phase potentially has a natural place in our model. It could occur if the temperature at which the monopoles become irrelevant is lower than the temperature at which charges become relevant, $T_{\rm BKT} < T_{\rm NC}$. Then between these two temperatures the theory in the infrared is the theory of massless photons. Indeed, consider a simple Z_N invariant theory of one phase field

$$\mathcal{L}_{\rm eff} = \frac{g^2}{8\pi^2 T}(\partial_\mu \phi)^2 + \xi \exp(iN\phi)\,. \tag{4.8}$$

We normalised the kinetic term so that for $N = 2$ the model reduces to the Polyakov effective theory for SU(2) GG theory. The BKT point in this theory is at

$$T_{\rm BKT} = \frac{2g^2}{\pi N^2}\,. \tag{4.9}$$

If the only vortices that are allowed have integer vorticity, the temperature at which they become relevant does not depend on N and is

$$T_{\rm NC} = \frac{g^2}{8\pi}\,. \tag{4.10}$$

Thus for $N > 4$ the "monopole binding" occurs prior to the "charge deconfinement" and there is an intermediate massless phase, bounded by two BKT transitions.

Let us analyse in more detail how the model eq. (3.8) behaves when one photon is much lighter than the others. The simplest case is SU(3) eq. (2.28). Let us take W_1 to be lighter than W_2 and W_3. This means that in eq. (2.28) we have $\xi_1 \gg \xi_2, \xi_3$. To minimise the first term in the potential, dynamically the difference of the phases of the two vortex fields must be constant. Thus on the low energy states we have

$$V_1 = V_2\,. \tag{4.11}$$

With this identification we indeed get the theory of one phase field. However the coefficient of the kinetic term is "renormalised" due to the off diagonal form of A_{ij}. In this case we find

$$\mathcal{L}_{\rm eff} = \frac{3g^2}{8\pi^2 T}(\partial_\mu \chi)^2 + \xi \exp(i3\chi)\,. \tag{4.12}$$

This reduction procedure is easily extended to any N. One can always choose appropriate W's to be light, so that at low energy *all* vortex fields become equal

$$V_i = V_j\,. \tag{4.13}$$

The effective theory then is

$$\mathcal{L}_{\rm eff} = \frac{N(N-1)g^2}{16\pi^2 T}(\partial_\mu \chi)^2 + \xi \exp(iN\chi)\,. \tag{4.14}$$

Interestingly, the coefficient of the kinetic term of the only remaining field is of order N^2, which is the number of degrees of freedom in the underlying Yang-Mills theory. Thus the first thing to note is that the BKT temperature does not decrease as suggested by eq. (4.9), but rather increases with N as

$$T_{\rm BKT} = \frac{g^2(N-1)}{\pi N} \tag{4.15}$$

so that at $N \to \infty$ its value is twice that of $N = 2$.

To calculate $T_{\rm NC}$ we should look at the terms that contain dual fields in eq. (3.8). The structure of the phases in these terms is exactly the same as the structure of the phases in the monopole induced term. Thus clearly taking all χ_i (and therefore $\tilde{\chi}_i$) to be equal some of these phases will vanish, while others will give the only surviving $\tilde{\chi}$ field with the coefficient N. Thus the charge terms reduce to

$$\mu \exp\left(i\frac{g^2}{4\pi T}N\tilde{\chi}\right)\,. \tag{4.16}$$

We then easily get

$$T_{\rm NC} = \frac{g^2 N}{16\pi(N-1)}\,. \tag{4.17}$$

So $T_{\rm NC}$ decreases with N. Perhaps surprisingly, we therefore find that as N becomes larger the two temperatures never cross, and in fact the difference between them grows. Nevertheless the temperature at which the scaling dimensions of the two operators are equal always stays equal to the geometrical mean of the two temperatures $\frac{g^2}{4\pi}$, in exact agreement with the analysis in the full theory eq. (3.8).

Why does this happen? If we were to allow only the vortices that preserve the condition $\chi_i = \chi_j$, the only perturbations involving the dual fields would be of the form $\mu \exp(i\frac{g^2}{4\pi T}N(N-1)\tilde{\chi})$. This indeed would lead to much higher $T_{\rm NC}$ so that for $N > 4$ the $T_{\rm BKT}$ and $T_{\rm NC}$ would cross. However the lagrangian eq. (3.8) contains perturbations which create vorticity of a single phase field χ_i, and thus effectively violate the equality of all phases. Another way of looking at it is to think of the field χ in eq. (4.14) as the average field $\chi = \sum_{i=1}^{N-1} \chi_i / N - 1$. The perturbations in eq. (3.8) then induce fractional vorticity $2\pi/(N-1)$. The corresponding operators are more relevant than those with vorticity one and thus the temperature $T_{\rm NC}$ is lower than one would naively expect. This effect is obviously due to the presence of the $N-1$ independent fields all of which can be excited independently. Thus even though at low temperature the effective theory had only one light field, all fields are important in the transition region.

The preceding discussion is of course only illustrative, since it neglects the effects of the lightest W-bosons. Those light bosons lead to large monopole fugacity $\xi = \exp\{-4\pi M_W/g^2\}$, which has an effect of freezing some of the phases of the vortex fields. However at finite temperature it is these same W-bosons which are produced more copiously than the others due to their relatively large fugacity $\mu = \exp\{-M_W/T\}$. The appearance of these W-bosons however tends to disorder precisely the same phase fields which are frozen by the corresponding monopole term by imposing non-vanishing vorticity on them. Thus the behaviour of the theory at criticality will be strongly affected by the presence of these particles and can not be directly deduced from the effective theory of only one scalar field, even allowing for fractional vorticity.

It is interesting to note, that if we go high enough above the critical temperature where the monopole terms are irrelevant and can be neglected, the theory is described again quite well in terms of one light field. In this regime the large fugacity of light W's leads to dynamical constraint $\tilde{\chi}_i = \tilde{\chi}_j$ and we have the theory of one light dual field.

5. Relating to pure Yang-Mills

Although our analysis is not directly relevant to pure Yang-Mills theory, it can be cast in the form which suggests that the relation exists and indeed may be closer than apparent at the first glance.

The high energy phase of the Yang-Mills theory is indeed customarily described in terms of $N-1$ light fields. Those are the phases associated with the eigenvalues of the Polyakov loop, P [20]. Since P is a special unitary matrix, it has $N-1$ independent eigenvalues. In fact these phases — the components of scalar potential A_0, are directly related to the dual fields $\tilde{\eta}_i$ of eq. (3.8) [15]. The dual fields $\tilde{\eta}_i$ appear in the last term of eq. (3.8). This term is nothing but the free energy of the charged particles W. This free energy is usually expressed in terms of P. In the regime where the Higgs expectation value is large and W's are heavy, the only light components of the vector potential are the diagonal ones. Hence in this regime the Polyakov loop is naturally diagonal. The free energy of a charged particle with the set of abelian charges $\vec{\alpha}$ is then given by the product of the appropriate eigenvalues of P. Comparing this with the last term of eq. (3.8) we have

$$A_0^i = \frac{g^2}{4\pi T}\vec{w}_i\vec{\tilde{\eta}}\,, \tag{5.1}$$

where A_0^i is the phase of the i-th eigenvalue of P. Remembering the the following relations between the roots and the weights of SU(N)

$$\vec{\alpha}_{ij} = \vec{w}_i - \vec{w}_j\,, \qquad \sum_{i=1}^{N} w_i^a w_i^b = \frac{1}{2}\delta^{ab} \tag{5.2}$$

we can rewrite the effective lagrangian in the hot phase (where the monopole terms are irrelevant) as

$$\frac{T}{g^2}\sum_{i=1}^{N}(\partial_\mu A_0^i)^2 + \sum_{ij}\mu_{ij}\cos(A_i^0 - A_j^0)\,. \tag{5.3}$$

The phases $\exp i\{A_0^i - A_0^j\}$ are eigenvalues of $\mathcal{P}$, where $\mathcal{P}$ is the Polyakov loop in the adjoint representation. Equation (5.3) is therefore of the form similar to the "effective action" discussed in the framework of hot QCD. Thus the "effective potential" in our case is given by a linear combination of the eigenvalues of the adjoint Polyakov loop. In fact, at the fixed point where all the fugacities are equal interestingly enough the potential term generated by W — can be written simply as

$$\mu\,\mathrm{Tr}\,\mathcal{P}\,. \tag{5.4}$$

In the hot Yang-Mills theory on the other hand the effective potential is given by the Bernoulli polynomial [20]. The origin of this difference is of course the large mass of W-bosons in the GG model. The partition function of a heavy charged particle is well approximated by the Polyakov loop. Our derivation corresponds to the leading term in the low temperature expansion (expansion in powers of the Boltzmann factor) which in the GG model is valid even far above the critical temperature. In pure Yang-Mills on the other hand the "charged particles" — gluons, are massless. As a result the particles are relativistic and their partition function is not given by the Polyakov loop. Also the low temperature as such does not exist, and the standard perturbative calculation corresponds to the genuine high temperature expansion.

Nevertheless it is interesting to observe, that some quantities calculated in the GG model behave in a way very similar to that in QCD. In particular consider the ratio of the longest correlation length in the sectors with total vorticity k, $1/m_k$ to that of vorticity 1. By total vorticity we mean the quantum number with respect to the magnetic Z_N symmetry. This correlation length can be extracted from the correlation functions of products of k vortex operators $\langle V_{i_1}\cdots V_{i_k}\rangle$. In general this calculation is quite laborious since the different vortex operators are not degenerate.

However they do become degenerate on the trajectory leading to the fixed point, where all the fugacities are equal. As explained in [4, 15] at high temperature the inverse correlation length in the vortex channel is given by the "wall tension" of the Z_N domain wall — solution of the equations of motion for the fields A_0^i with boundary conditions

$$\exp\{iA_0^i(x)\} \longrightarrow_{x\to-\infty} 1\,, \qquad \exp\{iA_0^i(x)\} \longrightarrow_{x\to\infty} \exp\left\{\frac{i2\pi k}{N}\right\}, \tag{5.5}$$

where x is the coordinate transverse to the "wall". In the pure Yang-Mills theory the result of this calculation is [21]

$$\frac{m_k}{m_1} = \frac{k(N-k)}{N-1}\,. \tag{5.6}$$

The equations of motion for the lagrangian eq. (5.3) are (we take all variables to depend only on one coordinate)

$$\frac{2T}{g^2}\frac{d^2}{dx^2}(A_0^i - A_0^N) + \sum_{j\neq i}\mu_{ij}\sin[A_0^i - A_0^j] - \sum_{j\neq N}\mu_{Nj}\sin[A_0^N - A_0^j] = 0\,. \tag{5.7}$$

We are unable to solve these equations in the general case. However in two special cases they are easy to analyse. Consider first the case discussed in the previous subsection, when only one of the fields A_0^i is light. Then obviously on the solution we must have $A_0^i = A_0^j = A$, $i,j, = 1,\ldots,N-1$. Since A_0^i are phases of the eigenvalues of the special unitary matrix, the last component must then be $A_0^N = (1-N)A$. These relations must hold for the solution with any k, including $k=1$. Then A satisfies the equation

$$\frac{2TN}{g^2}\frac{d^2}{dx^2}A + \mu\sin[NA] = 0 \tag{5.8}$$

with $\mu = \sum_{j\neq N}\mu_{Nj}$. This is the equation for one scalar field with potential $\cos[NA]$. In this case clearly as long as $k \leq N-k$, the solution for $k\neq 1$ is just the set of k well separated solutions for $k=1$. When $k \geq N-k$, the same boundary condition eq. (5.5) can be satisfied by having $N-k$ walls. Thus the tension of the k-fold wall is

$$m_k = \min\{k, N-k\}m_1\,. \tag{5.9}$$

The other simple case is when all the fugacities are degenerate. Then following [21] we can try the following ansatz for solution

$$\begin{aligned} A_0^i &= kA\,, \qquad i = 1,\ldots,N-k\,,\\ A_0^i &= (k-N)A\,, \qquad i = N-k+1,\ldots,N\,. \end{aligned} \tag{5.10}$$

The resulting equation for A is

$$\frac{2T}{g^2}\frac{d^2}{dx^2}A + \mu\sin[NA] = 0\,. \tag{5.11}$$

This does not depend on k. The tension for such a solution scales as does the kinetic term [21] as $k(N-k)$. Thus the wall tension and the inverse correlation length in the channel with vorticity k scales like in hot Yang-Mills theory according to eq. (5.6).

Thus even though generically the ratio m_k/m_1 in the GG model is not universal, and depends on the details of the masses of the W-bosons, close to criticality it follows exactly the same simple formula as in hot QCD.

We can analyse in precisely the same way the behaviour of the ratios of the string tensions of k-strings below the transition temperature. Due to the self duality of the fixed point, the effective lagrangian in terms of the phases of the vortex operators χ_i is identical to the lagrangian for A_0^i with the substitution $\mu \to \zeta$, $4\pi T/g^2 \to \frac{g^2}{4\pi T}$. The tension of the confining string is then calculated as the tension of the domain wall separating vacua with different values of χ_i [2]. We thus find that the ratios of the string tensions also follow the relation eq. (5.6). In fact this scaling relation is commonly known under the name of "Casimir scaling" and is observed to hold for the ratios of the string tensions in pure Yang-Mills theory at low temperature [22] in both four and three dimensions.

6. Conclusions

An interesting feature of our result is that the critical temperature in the SU(N) theory at large N is proportional to the coupling g^2 and not to 't Hooft coupling $\lambda = g^2 N$. Thus at large N the critical temperature approaches zero. The physical reason for this is easy to understand. At large N and fixed λ the Higgs VEV should also scale with N in such a way that the mass of W-bosons remains fixed. The monopole action then grows as N and the photons get progressively lighter (exponentially with N).[4] Thus the thickness of the confining string grows and the density of W-bosons needed to restore the symmetry becomes smaller and smaller.

More importantly, our main conclusion is that the deconfining transition in the SU(N) GG model is second order and the universality class is determined by the infrared fixed point eq. (4.7). This point is Z_N symmetric and self dual. We have given some arguments supporting the possibility that the fixed point theory is the Z_N parafermionic model [18] although we were not able to prove this explicitly. We can however definitely exclude Potts and Villain universality classes. In this context we also note that the ratios of the "wall tensions" calculated in the previous section (eq. (5.6)) for $N > 3$ are different from the corresponding ratios in Villain model

[4]This is analogous to the situation in QCD where the instantons become less relevant at large N and the η' meson becomes massless. The major difference is of course that while the η' mass in QCD decreases as $1/N$, the photon masses in GG model decrease exponentially. This difference is due to the non diluteness of the instanton gas in QCD as opposed to diluteness of the monopole gas in the GG model.

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(which follow eq. (5.9) as well as in Potts model (where all the tensions are equal $m_k = m_1$).[5] This again tentatively supports our expectation that the universality class of the GG model is different.

To answer this question one should study (numerically or analytically) the class of Z_N invariant spin systems which has not been studied so far. The lagrangian of the relevant model can be taken as eq. (2.26). This is an explicit lagrangian of $N-1$ interacting phase fields which can be easily discretized to define a lattice Z_N invariant spin system. Hopefully the W_N symmetry of the $\mathrm{SU}(N)_1$ WZNW model can be of help here too.

Interestingly, contrary to naive universality arguments the transition is neither first order as in the N-state Potts model, nor in the U(1) universality class as in the Villain model. We believe the reason is precisely the large number of light fields present in the theory. It is well known that oftentimes the symmetry alone does not fix the universality class of the transition, the number of light fields being the other important element.

An interesting question is of course what happens in the pure Yang-Mills theory. The global symmetry associated with the phase transition is still Z_N [4]. The crucial question is what is the number of light degrees of freedom. We think there is some grounds to believe that the description presented in this paper is relevant in this case too.

As discussed in the previous section there is direct correspondence in the hot phase between the light fields in the GG model and in the pure Yang-Mills theory. Again, the usual lore is that the behaviour of these same fields A_0 at critical temperature determine the universality class of the transition. Moreover, the ratios of the vortex correlation lengths as well as string tensions close to criticality in the GG model seems to be similar to pure Yang-Mills theory. This point of view would then fit with the proposition that the critical behaviour of the pure Yang-Mills theory is the same as that of the SU(N) GG model. Of course, universality arguments can never exclude the possibility of first order transition which can be forced upon the system by a heavy sector [23]. It would be interesting to investigate this question numerically by lattice gauge theory methods.

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[5] Technically speaking the calculation of the previous section is valid only far enough from criticality, so that the monopole terms could be neglected. We believe however that due to the self-duality of the fixed point the same behaviour will also survive in the critical region.

of spin systems and the 't Hooft/Wilson loop ratios. We also thank Andrea Cappelli for discussions about parafermions and Mike Teper and Biaggio Lucini for discussions on Casimir scaling.

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